Final Report to

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Problems in Fluid Dynamics

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With the assistance of this grant we completed our work on numerical gasdynamics. Of particular interest, we succeeded in developing a scheme for the parametric differentiation and integration of the gasdynamics equations. A numerical integration of the gasdynamic equations is necessarily carried out for a specific set of parameter values. Any flow depends on the many parameters which characterize geometry and flow conditions, such as aspect ratio, thickness ratio and Mach number. Typically a change in even one such parameter requires a new computation. However for purposes of performance studies, design or structural analysis one would like to have results for a continuous range of nearby parameter values. To treat this generally, one can consider the linear variational equations obtained by differentiating the exact equations with respect to each of the relevant parameters. The resulting matrix of derivatives of flow quantities is referred to as the Jacobi matrix. The subsequent procedure is then straightforward. One integrates the nonlinear governing equations, which results in the determination of just one point in parameter space, and simultaneously the variational equations governing the Jacobi matrix. The last is then used to determine the flow conditions for nearby parameter values.

The method has been tested for two dimensional supersonic flow post an airfoil, with airfoil thickness, camber and angle of attack varied. At

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Mach 2 the results for pressure distribution shows very good agreement between this approximate method and the exact results even as thickness is varied from 10% to 15%, camber varied from 0.0 to 0.2 or angle of attack varied from 0° to 10°. This approach has great potential value for rapidly assessing the effect of design changes.

The other focus of our work has been on problems in fluid stability, bifurcations and turbulence. One problem that we have tackled that relates chaotic dynamics to transition in shear flows is the flow behind a cylinder and the von Karman vortex trail used to represent the wake region. Studies of flows behind a bluff body in the frequency shedding range of Reynolds numbers have shown far more structure than previously detected. Sreeni-vasan has made measurements which show frequencies in addition to the sheeding frequency. In addition, 'windows of chaos' or pre-turbulence are found in these experiments. Some controversy exists about the origin of the additional oscillations and the consequent chaos. van Atta and Gharib contend that the oscillations are due to resonant oscillations of the cylinder. Further experiments will decide this issue. In any case, the presence of fluid oscillations and the chaos which is found is not in dispute. This aspect of the phenomenon is of theoretical and practical significance. Related studies of the same experimental arrangement have demonstrated simple devices

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for controlling the vortex shedding mechanism. Indeed using fairly small probes, heated and unheated, one can entirely suppress the vortices, and hence reduce the drag and delay separation.

On the theoretical side we have been able to account for the additional oscillations by showing the presence of vibrational modes in the von Karman vortex trail. In related investigations using the von Karman vortex model we have also been able to account for diverse effects such as vortex merging and vortex coupling. Both of these as well as other effects have been found in experiment and in computation. The phenomenon of wave propagation along the vortex trail was found by Tritton and we have recently verified this theoretically.

During this grant period we were able to initiate work on methods for turbulent data reduction and the determination of coherent structures. The direct numerical simulation of turbulent flows provides a wealth of information yet poses a challenge, just by the sheer volume of data, of how best to use this information. Further, experiment and flow visualization suggest that in many turbulent flows the flow is not entirely random but contains a large degree of structure over a range of length scales. An important objective is to then see how much information about coherent structures may be extracted from simulation data, and to investigate as far as possible

the nature and dynamics of these structures. Our approach is based on an eigenfunction analysis of turbulent velocity correlation tensor, and is also variously known as the Karhunen-Loeve expansion or Proper Orthogonal Decomposition.

In our research work that has continued after the expiration of this grant we have applied these methods successfully to the Ginzburg-Landau equation and to Benard convection between horizontal plane boundaries. The Ginzburg-Landau equation is a model problem, involving a partial differential equation is (x, t), which arises in the study of the nonlinear transitions of various shear flows. Under certain conditions it exhibits chaotic solutions. Analysis of these chaotic solutions by this eigenmode approach has shown that the dynamics can be represented by just a few eigenmodes and the chaos analyzed in terms of the evolution of these eigenmodes. Similarly for the turbulent thermal convection between horizontal plane boundaries, we find that most of the energy containing motion can be represented by relatively few modes. This approach has great potential in both reducing simulation data to manageable form and to probing the structure of these flows.

Publications

P.K. Daripa and L. Sirovich, J. Comp. Phys. 62 400-413, 1986. Exact and approximate gas dynamics using the tangent gas.

P.K. Daripa and L. Sirovich, J. Comp. Phys. 63, 311-328, 1986. An inverse method for subcritical flows.

Y-K Kwok and L. Sirovich, SIAM J. Appl. Math. 47, 279-295, 1987. On some aspects of the transonic controversy.

H.T. Sharp and L. Sirovich, submitted for publication. On constructing a continuous parameter range of computation flows.

Contributed Papers

C.C. Lim and L. Sirovich, Annual Div. of Fluid Dyn. meeting, Amer. Phys. Soc., Columbus, Ohio, 23-25 Nov. 1986. Dynamics of the von Karman Vortex Street.

H.T. Sharp and L. Sirovich, Annual Div. of Fluid Dyn. meeting, Amer. Phys. Soc., Columbus, Ohio, 23-25 Nov. 1986. Construction of Continuous Parameter Range of Computational Flows.